

BONN-TH-2005-03
hep-ph/0508150

Flavor-singlet hybrid baryons may already have been discovered

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Abstract

The splittings between the spin 1/2 and spin 3/2 iso-singlet baryons $\Lambda_s(1405)$ and $\Lambda_s(1520)$, and their charmed counterparts $\Lambda_c(2593)$ and $\Lambda_c(2625)$, have been a theoretical conundrum. Here we investigate the possibility that the QCD binding of color octets comprised of three quarks in a flavor singlet configuration is stronger than previously envisaged, allowing these states to be interpreted as hybrids consisting of three quarks plus a valence gluon ($udsg$) and ($udcg$). A fit of their mass separation allows the mass prediction of the strange and charmed flavor octet and decuplet hybrid baryons and the prediction of the mass separation of the beauty hybrids. Such hybrid states come in parity-doubled pairs with the even parity state lighter by about 300 MeV. Existing data accommodates either parity assignment for the observed states and the existence of the required unobserved partners at either higher or lower mass. We discuss difficulties with and strategies for observing the other states under the two cases. A corollary of the strong-binding-in-flavor-singlet-channel hypothesis is that the H-dibaryon may be very long lived or stable with $m_H \lesssim 2$ GeV.

1 Introduction

The spectrum of baryons between 1 GeV and 2 GeV has been accurately measured experimentally [1] and in most cases detailed partial wave analyses have lead to the determination of their spins and parities. Nearly all known baryon resonances are well-described as three-quark states, however the isosinglets $\Lambda_s(1405)$ (total angular momentum $J = 1/2$) and $\Lambda_s(1520)$ ($J = 3/2$) are problematic in this interpretation [2]. If they are orbital momentum $L = 1$ states in the three-quark model, their large energy splitting of 115 MeV cannot be explained by QCD calculations [3], which predict the two states to be nearly degenerate. Calculations which include spin-orbit interactions even predict an inversion of the masses [4], contrary to fact. An alternative suggestion is that the $\Lambda_s(1405)$ is a \bar{K} - N s -wave bound state [5]. Since the \bar{K} - N p -wave system would not bind, this interpretation does not provide any $J = 3/2$ partner, so the $\Lambda_s(1520)$ is taken to be a $L = 1$ three-quark state, and its $J = 1/2$ partner is assumed to be hiding, close in mass according to [3]. However this is also problematic, since in \bar{K} - N experiments this mass region is well explored without evidence of such a resonance [2].

QCD predicts other composite particles, color singlet states not only involving quarks but also “constituent” gluons [6]. The phenomenology of these baryons, called hybrids, was developed by Barnes and Close [7, 8], Golowich et al. [9] and others. These works use the bag model and the potential model to suggest that the lightest hybrid masses are below 2 GeV. Although there is evidence in favor of some candidates [10], no hybrids have yet been confirmed. One problem is that the mixing of hybrids with conventional states makes them hard to discriminate from normal states. Since the quantum numbers of hybrid states sometimes coincide with those of known particles, it is also possible that some octet and decuplet states identified as conventional three-quark baryon states could be hybrids [11, 12, 13].

Here we explore the possibility that the observed $J = 1/2$ and $J = 3/2$ isosinglet baryons $\Lambda(1405)$ and $\Lambda(1520)$ are actually hybrid baryon states [14]. To test the hypothesis, we calculate their mass splittings in the hybrid picture and find the correct amount (≈ 100 MeV) and ordering ($m_{3/2} > m_{1/2}$). Since the direct determination of the parity of $\Lambda(1405)$ is experimentally delicate and yet unclear [2], we consider both possibilities for its parity. A constituent gluon combining with three quarks in an $L = 0$ orbital ground state can form both parity even and parity odd states, with the latter about 300 MeV lighter [15]. Remarkably, both parity assumptions for the hybrids are compatible with existing data.

The paper is organized as follows. In Section 2 we review the general structure of hybrid baryon wave functions. We define the degeneracy lifting hyperfine interaction using effective hyperfine couplings between the constituents. In Section 3 we determine the quark-quark couplings from ordinary baryon mass splittings and calculate the flavor singlet hybrid mass splittings. We then use the observed flavor singlet splittings to fix the quark-gluon effective hyperfine coupling. This allows predictions to be made for octet and decuplet hybrid baryon mass splittings. In Section 4 we obtain the hyperfine couplings in the mesonic sector and relate them to the values found for baryonic states. In Sections 5 and 6 two implications of this model are discussed – parity doubling and a possible light H-dibaryon. Section 7 gives a summary of our results and conclusions.

A preliminary description of this work was reported in [16]. Since then, the experimental viability of a stable H has been established [17, 18, 19], making this scenario more

Table 1: Decomposition of $SU(18)$ in representations of color, flavor and spin.

$SU(18)$	$SU(3)_C$	$SU(3)_F$	$SU(2)_S$	# of these
816				16
				40
				160
				40
				128
				256
				160
				16

credible. In addition, the possible discovery of pentaquark states in the 1.5 GeV range underlines the need to be open-minded regarding the validity of naive QM dynamical assumptions.

2 Definitions and formalism

2.1 Wave functions of hybrid baryons

A systematic group theoretical classification of the hybrid wave functions is possible, as the quarks have to obey the Pauli principle. We consider systems of three quarks in the orbital ground state which have up to three different quark flavors u , d and i with $i = s, c, b$. The flavor group is $SU(3)_F$ if flavor symmetry is assumed, i.e. the mass differences between quarks of different flavors are neglected. The groups for color and spin are $SU(3)_C$ and $SU(2)_S$, respectively. A single quark is then a 18-dimensional representation of the direct product group $SU(3)_C \times SU(3)_F \times SU(2)_S$. We reduce the direct product of the three quarks in irreducible representations of $SU(18)$ with the help of Young tableaux [20]

$$\begin{matrix} \square & \times & \square & \times & \square \\ 18 & & 18 & & 18 \end{matrix} = \begin{matrix} \text{1140} \\ 1140 \end{matrix} + \begin{matrix} \text{1938} \\ 1938 \end{matrix} + \begin{matrix} \text{1938} \\ 1938 \end{matrix} + \begin{matrix} \text{816} \\ 816 \end{matrix}. \quad (1)$$

All physical qqq -states which obey the Pauli principle are members of the completely antisymmetric representation **816**. The decompositions of the multiplet into representations of $SU(3)_C$, $SU(3)_F$ and $SU(2)_S$ are given in Tab. 1. The gluon is in the color octet representation of $SU(3)_C$. The hybrid has to be a color singlet, thus the quarks have to be in the complex conjugate representation, which is again an octet. We will distinguish the qqq color octet states in the lower half of Tab. 1 by the shorthand *spin flavor*: **28**, **48**, **210**, **21**. There are altogether 70 color octet states with mixed symmetry which

form a $SU(6)$ representation of $SU(3)_F \times SU(2)_S$

$$\begin{array}{ccccccccc} \square & \times & \square & \times & \square & = & \square\Box\Box + \Box\Box + \Box\Box + \Box\Box \\ \textbf{6} & \quad \textbf{6} & \quad \textbf{6} & \quad \textbf{56} & \quad \textbf{70} & \quad \textbf{70} & \quad \textbf{20} \end{array}. \quad (2)$$

A Young tableau of the form $\Box\Box$ is mixed symmetric because the wave function which is assigned to the tableaux is symmetric (or antisymmetric) only under the exchange of two quarks. We choose them to be the first and second quark and will use this convention throughout. Since this choice is arbitrary, we give the wave functions in flavor space in Appendix B. We label the wave function φ_{MS} for mixed symmetric states and φ_{MA} for mixed antisymmetric states. A totally symmetric (totally antisymmetric) function $\phi_{S(A)}$, under the interchange of any two quarks, can be built out of mixed symmetric functions $\varphi_{MS(A)}$ and $\varphi'_{MS(A)}$ [15]

$$\phi_S = \frac{1}{\sqrt{2}}(\varphi_{MS}\varphi'_{MS} + \varphi_{MA}\varphi'_{MA}), \quad (3)$$

$$\phi_A = \frac{1}{\sqrt{2}}(\varphi_{MS}\varphi'_{MA} - \varphi_{MA}\varphi'_{MS}). \quad (4)$$

Mixed symmetric functions ϕ_{MS} and mixed antisymmetric functions ϕ_{MA} are [15]

$$\phi_{MS} = \frac{1}{\sqrt{2}}(-\varphi_{MS}\varphi'_{MS} + \varphi_{MA}\varphi'_{MA}), \quad (5)$$

$$\phi_{MA} = \frac{1}{\sqrt{2}}(\varphi_{MS}\varphi'_{MA} + \varphi_{MA}\varphi'_{MS}). \quad (6)$$

The four color octet wave functions, which lie in the **816** of $SU(18)$, are then [15]

$$^4\mathbf{8} : \psi_A = \frac{1}{\sqrt{2}}(c_{MS}f_{MA} - c_{MA}f_{MS})s_S, \quad (7)$$

$$^2\mathbf{10} : \psi_A = \frac{1}{\sqrt{2}}(c_{MSSMA} - c_{MASMS})f_S, \quad (8)$$

$$^2\mathbf{8} : \psi_A = \frac{1}{2}[c_{MS}(f_{MASMS} + f_{MSSMA}) - c_{MA}(f_{MASMA} - f_{MSSMS})], \quad (9)$$

$$^2\mathbf{1} : \psi_A = \frac{1}{\sqrt{2}}(c_{MASMA} + c_{MSSMS})f_A. \quad (10)$$

with $\varphi = f, s, c$ for flavor, spin and the color three quark wave function, respectively. We will include flavor breaking and work in the approximation that isospin is exact. In that case, we distinguish between isospin singlet ($I = 0$) and isospin triplet ($I = 1$) hybrid baryons

$$^4\mathbf{8}(I = 0, 1), \quad ^2\mathbf{10}(I = 1), \quad ^2\mathbf{8}(I = 0, 1), \quad ^2\mathbf{1}(I = 0). \quad (11)$$

These states combine with the gluon, which thus gains effective mass [8, 9] and a third degree of freedom, the spin zero state. The confined gluon is classified by a TE or TM mode [15, 21], with total angular momentum J and parity P

$$TE : J^P = 1^+, 2^-, \dots \quad TM : J^P = 1^-, 2^+, \dots. \quad (12)$$

In the bag model, the TE $J^P = 1^+$ mode is estimated to be about 300 MeV lighter than the TM $J^P = 1^-$ mode [15].

2.2 Interaction Hamiltonian

We divide the hyperfine interaction Hamiltonian V_{hyp} into interactions only between the quarks V_{qq} , and interactions between the quarks and the constituent gluon V_{qg}

$$V_{hyp} = V_{qq} + V_{qg}. \quad (13)$$

With the effective hyperfine couplings κ_{ij} between the quarks, we define the interaction

$$V_{qq} = - \sum_{i < j} \kappa_{ij} \mathbf{S}^i \cdot \mathbf{S}^j \mathbf{F}^i \cdot \mathbf{F}^j, \quad (14)$$

which has the color-spin structure of 1-gluon exchange [9]. The spin matrices \mathbf{S}^i and color matrices \mathbf{F}^i for the i th quark are

$$S_m^i = \frac{1}{2}\sigma_m, \quad F_a^i = \frac{1}{2}\lambda_a, \quad (15)$$

where σ_m , $m = 1, 2, 3$, and λ_a , $a = 1, \dots, 8$, are the Pauli and the Gell-Mann matrices [15], and the matrix products are defined by

$$\mathbf{S}^i \cdot \mathbf{S}^j = \sum_{m=1}^3 S_m^i S_m^j, \quad \mathbf{F}^i \cdot \mathbf{F}^j = \sum_{a=1}^8 F_a^i F_a^j. \quad (16)$$

The effective one-gluon ansatz leads to the quark-gluon interaction

$$V_{qg} = - \sum_i \kappa_{ig} \mathbf{S}^i \cdot \mathbf{S}^g \mathbf{F}^i \cdot \mathbf{F}^g, \quad (17)$$

where \mathbf{S}^g and \mathbf{F}^g are the gluon spin and color matrices. The Hamiltonian is then

$$V_{hyp} = - \sum_{i < j} \kappa_{ij} \mathbf{S}^i \cdot \mathbf{S}^j \mathbf{F}^i \cdot \mathbf{F}^j - \sum_i \kappa_{ig} \mathbf{S}^i \cdot \mathbf{S}^g \mathbf{F}^i \cdot \mathbf{F}^g. \quad (18)$$

We label the effective coupling between two light quarks by κ and those between a light and a heavy quark by κ_i , with $i = s, c, b$ the index for the heavy quark. The coupling between a gluon and a light quark is labeled by κ_g and those between a gluon and a heavy quark is labeled by κ_{ig} . The hierarchy of the κ 's follows from the form of the “Fermi-Breit-interaction” in QCD for single gluon exchange [13]

$$V^{ij} \propto \frac{\mathbf{F}^i \cdot \mathbf{F}^j \mathbf{S}^i \cdot \mathbf{S}^j}{m_i m_j}. \quad (19)$$

The effective coupling κ would therefore be inversely proportional to the product of the masses of the interacting particles

$$\frac{\kappa_i}{\kappa_j} = \frac{m_j}{m_i}. \quad (20)$$

Moreover, the hyperfine coefficient (19) is the product of the color magnetic moments. Thus replacing a light quark color magnetic moment with a gluon magnetic moment we could determine κ_g from κ . Since this is the same replacement independent of quark flavor, we expect

$$\frac{\kappa}{\kappa_g} = \frac{\kappa_s}{\kappa_{sg}} = \frac{\kappa_c}{\kappa_{cg}}. \quad (21)$$

These relations reduce the number of parameters in the fit of hybrid baryons and makes it more predictive. We determine the coupling strengths κ and κ_i from ordinary baryon mass splittings in Section 3.1 and κ_g and κ_{ig} in Section 3.2.

2.3 Quark-quark interactions

We calculate the matrix elements $\langle V_{qq} \rangle$ (14) of the interaction Hamiltonian (13), including flavor breaking and assuming that isospin is conserved. An operator O for a baryon with two light quarks $q = u, d$ and one heavy quark $i = s, c, b$ has then the structure $O = O^S + \epsilon O^{12}$. The part O^S is flavor symmetric and symmetric under the interchange of any two quarks. The part O^{12} is isospin symmetric and symmetric under the interchange of the light quarks 1 and 2 only. The measure for flavor breaking is the mass difference between the heavy and the light quark. In the limit of unbroken flavor symmetry, ϵ goes to zero. Such a decomposition of V_{qq} leads to

$$V_{qq} = -\kappa_3 \sum_{i < j} \mathbf{S}^i \cdot \mathbf{S}^j \mathbf{F}^i \cdot \mathbf{F}^j - (\kappa - \kappa_3) \mathbf{S}^1 \cdot \mathbf{S}^2 \mathbf{F}^1 \cdot \mathbf{F}^2. \quad (22)$$

The first term of (22) is completely symmetric. Its evaluation for exact flavor symmetry leads to [22]

$$\langle \sum_{i < j} \mathbf{S}^i \cdot \mathbf{S}^j \mathbf{F}^i \cdot \mathbf{F}^j \rangle = \frac{21}{16} - \frac{1}{8} C_C^{qqq} - \frac{1}{4} C_F^{qqq} - \frac{1}{12} C_S^{qqq}, \quad (23)$$

with $C_{C,F,S}^{qqq}$ the Casimir operators of the three quarks for color, flavor and spin, respectively, as defined in Appendix A. Alternatively, this term can be evaluated using the $SU(6)$ Casimir operators C_6^{qqq} of color and spin of the three quarks [23]

$$\langle \sum_{i < j} \mathbf{S}^i \cdot \mathbf{S}^j \mathbf{F}^i \cdot \mathbf{F}^j \rangle = \frac{1}{32} C_6^{qqq} - \frac{1}{12} C_S^{qqq} - \frac{1}{8} C_C^{qqq} - \frac{3}{2}. \quad (24)$$

The definition and values of C_6 in various representations may be found in [24]. We give the values of $\langle \sum_{i < j} \mathbf{S}^i \cdot \mathbf{S}^j \mathbf{F}^i \cdot \mathbf{F}^j \rangle$ for the color octet three quark states in the first part of Tab. 2.

Table 2: Expectation values of $O^A = \sum_{i < j} \mathbf{S}^i \cdot \mathbf{S}^j \mathbf{F}^i \cdot \mathbf{F}^j$ and $O^{12} = \mathbf{S}^1 \cdot \mathbf{S}^2 \mathbf{F}^1 \cdot \mathbf{F}^2$

$SU(3)_C$	8	8	8	8
$SU(3)_F$	8	8	10	1
$SU(2)_S$	2	4	2	2
$\langle O^A \rangle$	1/8	-1/8	-5/8	7/8

$SU(3)_C$	3	3	6	6
$SU(2)_I$	1	3	1	3
$SU(2)_S$	1	3	3	1
$\langle O^{12} \rangle$	1/2	-1/6	1/12	-1/4

The second term of (22) is symmetric under interchange of quark 1 and 2. Using isospin rather than flavor, we can modify (23) for this case and find

$$\langle \mathbf{S}^1 \cdot \mathbf{S}^2 \mathbf{F}^1 \cdot \mathbf{F}^2 \rangle = \frac{2}{3} - \frac{1}{8} C_C^{12} - \frac{1}{4} C_I^{12} - \frac{1}{12} C_S^{12}. \quad (25)$$

$C_{C,I,S}^{12}$ are the Casimir operators of the quark 1 and 2 for color, isospin and spin, respectively. We give the values for $\langle \mathbf{S}^1 \cdot \mathbf{S}^2 \mathbf{F}^1 \cdot \mathbf{F}^2 \rangle$ for all possible antisymmetric representations in flavor, color and isospin of the diquark in the second part of Tab. 2.

2.4 Quark-gluon interactions

The quark-gluon interaction (17) also decomposes into two terms

$$V_{qg} = -\kappa_{3g} \sum_i \mathbf{S}^i \cdot \mathbf{S}^g \mathbf{F}^i \cdot \mathbf{F}^g - (\kappa_g - \kappa_{3g})(\mathbf{S}^1 \cdot \mathbf{S}^g \mathbf{F}^1 \cdot \mathbf{F}^g + \mathbf{S}^2 \cdot \mathbf{S}^g \mathbf{F}^2 \cdot \mathbf{F}^g). \quad (26)$$

The first term is completely symmetric under interchange of any pair of quarks and the second term is symmetric only under interchange of quark 1 and 2. We list our results for the flavor singlet and the results for the light octets and decuplets [8] in Tab. 3.

Table 3: Values for $\langle \sum_i \mathbf{S}^i \cdot \mathbf{S}^g \mathbf{F}^i \cdot \mathbf{F}^g \rangle$ and $\langle \mathbf{S}^1 \cdot \mathbf{S}^g \mathbf{F}^1 \cdot \mathbf{F}^g \rangle$

<i>spin flavor</i>	48			210		28		21		
total J	5/2	3/2	1/2	3/2	1/2	3/2	1/2	3/2	1/2	
$\langle \sum_i \mathbf{S}^i \cdot \mathbf{S}^g \mathbf{F}^i \cdot \mathbf{F}^g \rangle$	-3/2	1	5/2	0	0	-1/2	1	-1	2	
$\langle \mathbf{S}^1 \cdot \mathbf{S}^g \mathbf{F}^1 \cdot \mathbf{F}^g \rangle$		I=1			I=1		I=1		I=0	
$\langle \mathbf{S}^1 \cdot \mathbf{S}^g \mathbf{F}^1 \cdot \mathbf{F}^g \rangle$	-3/8	1/4	5/8	-1/4	1/2	-1/4	1/2	0	0	
	I=0			I=0		I=0				
	-5/8	5/12	25/24			0	0			

For the flavor octets and decuplets which contain only light quarks u and d , the second term of (26) vanishes because $\kappa_g = \kappa_{3g}$. For hybrids containing one heavy quark $i = s, c, b$ we use the symmetry of the wave function under interchange of quark 1 and 2

$$\langle \mathbf{S}^1 \cdot \mathbf{S}^g \mathbf{F}^1 \cdot \mathbf{F}^g \rangle = \langle \mathbf{S}^2 \cdot \mathbf{S}^g \mathbf{F}^2 \cdot \mathbf{F}^g \rangle, \quad (27)$$

and write

$$\begin{aligned} & \langle \mathbf{S}^1 \cdot \mathbf{S}^g \mathbf{F}^1 \cdot \mathbf{F}^g \rangle \\ &= \frac{1}{4} [(\mathbf{S}^1 + \mathbf{S}^g)^2 - (\mathbf{S}^1)^2 - (\mathbf{S}^g)^2] [(\mathbf{F}^1 + \mathbf{F}^g)^2 - (\mathbf{F}^1)^2 - (\mathbf{F}^g)^2] \\ &= \frac{1}{4} \left[(\mathbf{S}^1 + \mathbf{S}^g)^2 - \frac{11}{4} \right] \left[(\mathbf{F}^1 + \mathbf{F}^g)^2 - \frac{13}{3} \right]. \end{aligned} \quad (28)$$

What remains is to determine the spin and color representations of the gluon–first quark state ($\mathbf{S}^1 + \mathbf{S}^g$ and $\mathbf{F}^1 + \mathbf{F}^g$) of each hybrid baryon. In order to expand the color and spin wave function of each hybrid (with the help of $SU(2)$ [1] and $SU(3)$ [25] Clebsch Gordan coefficients) into gluon–first quark and second quark–third quark color and spin wave functions, we need to know the spin and color representations of the diquark. The 1-2 antisymmetric part of each hybrid wave function has to fulfill two constraints. Firstly, the flavor part must be f_{MS} (or f_S) for isotriplets and f_{MA} (or f_A) for isosinglets. Secondly, the wave function must be antisymmetric under interchange of quark 1 and 2. The wave functions for the **48** hybrids result immediately from (7)

$$\mathbf{48}(I=1) : f_{MS CMASS}, \quad (29)$$

$$\mathbf{48}(I=0) : f_{MACMSS}. \quad (30)$$

For the other hybrids states (8)-(10), the wave function of each hybrid may be a linear combination (l.c.) of the following functions

$$^2\mathbf{1}(I=0) : \text{l.c. of } f_{ACMSSMS} \text{ and } f_{ACMASMA}, \quad (31)$$

$$^2\mathbf{10}(I=1) : \text{l.c. of } f_{SCMSSMA} \text{ and } f_{SCMASMS}, \quad (32)$$

$$^2\mathbf{8}(I=1) : \text{l.c. of } f_{MSCMSSMA} \text{ and } f_{MSCMASMS}, \quad (33)$$

$$^2\mathbf{8}(I=0) : \text{l.c. of } f_{MACMSSMS} \text{ and } f_{MACMASMA}. \quad (34)$$

All the above parts of the wave functions are eigenfunctions of the operator

$$O^{12} = -(\kappa - \kappa_3) \mathbf{S}^1 \cdot \mathbf{S}^2 \mathbf{F}^1 \cdot \mathbf{F}^2 - 2(\kappa_g - \kappa_{3g}) \mathbf{S}^1 \cdot \mathbf{S}^g \mathbf{F}^1 \cdot \mathbf{F}^g, \quad (35)$$

which is the 1-2 symmetric part of our interaction Hamiltonian V_{hyp} (13). We assume that the diquark is in a state in which the energy is minimal, i.e., in which O^{12} is minimized. The wave functions with minimal energy are

$$^2\mathbf{1}(I=0) : f_{ACMASMA}, \quad (36)$$

$$^2\mathbf{10}(I=1) : f_{SCMASMS}, \quad (37)$$

$$^2\mathbf{8}(I=1) : f_{MSCMASMS}, \quad (38)$$

$$^2\mathbf{8}(I=0) : f_{MACMASMA}. \quad (39)$$

We list the resulting values for $\langle \mathbf{S}^1 \cdot \mathbf{S}^g \mathbf{F}^1 \cdot \mathbf{F}^g \rangle$ in Tab. 3, and values for $\langle \mathbf{S}^1 \cdot \mathbf{S}^2 \mathbf{F}^1 \cdot \mathbf{F}^2 \rangle$ can be taken from the second part of Tab. 2.

3 Mass splittings of hybrid baryons

3.1 Quark-quark couplings for baryons

In order to find values for the effective couplings κ , κ_s and κ_c we fit the mass splittings of the baryons Σ_i^* , Σ_i and Λ_i , labeled by the flavor index of the heavy quark $i = s, c, b$. In the case of exact $SU(3)_F \times SU(2)_S$ symmetry, the isospin multiplets Σ_i^* , Σ_i and Λ_i (for i fixed) would be members of the totally symmetric 56-dimensional representation, see (2). By switching on the mass difference between the heavy quark i and the light quarks q we break $SU(3)_F$. The values of κ , κ_s and κ_c can thus be determined by the experimentally observed mass separations of the isospin multiplets given in Tab. 4. Calculating the quark-quark interaction (22), we find for these mass splittings [15]

$$E(\Sigma_i^*) - E(\Sigma_i) = \kappa_i, \quad (40)$$

$$E(\Sigma_i) - E(\Lambda_i) = \frac{2}{3}\kappa - \frac{2}{3}\kappa_i, \quad (41)$$

$$E(\Sigma_i^*) - E(\Lambda_i) = \frac{2}{3}\kappa + \frac{1}{3}\kappa_i. \quad (42)$$

With increasing mass of the heavy quark i , κ_i decreases (20) so that the $\Sigma_i^* - \Sigma_i$ splittings decrease, the $\Sigma_i - \Lambda_i$ splittings increase and the $\Sigma_i^* - \Lambda_i$ splittings decrease.

In addition, we can predict the order of the Σ_b^* , Σ_b and Λ_b mass splittings, which have not yet been measured. Their mass splittings depend on κ_b which can be estimated with $m_c = 1.25$ GeV, $m_b = 4.25$ GeV [1] and relation (20)

$$\kappa_b = \frac{m_c}{m_b} \kappa_c = 18 \text{ MeV}. \quad (43)$$

We give the mass splittings of the beauty baryons in Tab. 4.

Table 4: The baryons used to fix the κ_i and predicted splittings of beauty baryons

κ_i	baryon	spin	isospin	flavor	content	ΔM [MeV]	fit	κ_i [MeV]
κ	$\Delta(1232)$	3/2	3/2	10	qqq	$\Delta - N = 293$	293	293
	$N(939)$	1/2	1/2	8	qqq			
κ_s	$\Sigma_s^*(1385)$	3/2	1	10	qqs	$\Sigma_s^* - \Sigma_s = 192$	182	182
	$\Sigma_s(1193)$	1/2	1	8	qqs	$\Sigma_s - \Lambda_s = 77$	74	
	$\Lambda_s(1116)$	1/2	0	8	qqs	$\Sigma_s^* - \Lambda_s = 269$	256	
κ_c	$\Sigma_c^*(2520)$	3/2	1	10	qqc	$\Sigma_c^* - \Sigma_c = 65$	60	60
	$\Sigma_c(2455)$	1/2	1	8	qqc	$\Sigma_c - \Lambda_c = 170$	155	
	$\Lambda_c(2285)$	1/2	0	8	qqc	$\Sigma_c^* - \Lambda_c = 235$	215	
κ_b	$\Sigma_b^*(?)$	3/2	1	10	qqb	$\Sigma_b^* - \Sigma_b = ?$	18	18
	$\Sigma_b(?)$	1/2	1	8	qqb	$\Sigma_b - \Lambda_b = ?$	183	
	$\Lambda_b(5640)$	1/2	0	8	qqb	$\Sigma_b^* - \Lambda_b = ?$	201	

3.2 Mass splittings of the flavor singlet hybrid baryons

If flavor octet or decuplet hybrid baryons had been identified, we could use them to determine the effective quark-gluon couplings κ_{ig} ; instead we use the ansatz that $\Lambda_s(1405)$ and $\Lambda_s(1520)$ are flavor singlet hybrid baryons. When the **21** three quark state couples to the constituent gluon (spin triplet), a $J = \frac{1}{2}$ and a $J = \frac{3}{2}$ hybrid state are formed. From (22), (26) and tables 2 and 3 we find a mass separation of

$$E_{hyp}(J = 3/2) - E_{hyp}(J = 1/2) = 3\kappa_{3g}. \quad (44)$$

For the strange and the charm system we have

$$\Lambda_s(1520) - \Lambda_s(1405) = 115 \text{ MeV} = 3\kappa_{sg} \quad (45)$$

$$\Lambda_c(2625) - \Lambda_c(2593) = 32 \text{ MeV} = 3\kappa_{cg}. \quad (46)$$

As will be seen below, we could alternatively fix one difference and predict the other. The values for κ_{sg} and κ_{cg} are

$$\kappa_{sg} = 38 \text{ MeV} \quad (47)$$

$$\kappa_{cg} = 11 \text{ MeV}. \quad (48)$$

Using these values for κ_{sg} , κ_{cg} and relation (21) we find for $i = s$ and $i = c$ respectively, $\kappa_g = 61$ MeV and $\kappa_g = 54$ MeV. The similarity of these values is a check on the validity of the hybrid ansatz. The average value of

$$\kappa_g = (58 \pm 4) \text{ MeV}, \quad (49)$$

with a relative error less than 10% is sufficient for our effective model to be predictive.

Finally, to estimate the mass difference of $\Lambda_b(J = 3/2)$ and $\Lambda_b(J = 1/2)$, we use (20), (21) and $m_c = 1.25$ GeV, $m_b = 4.25$ GeV [1] to estimate

$$\kappa_{bg} = \frac{m_c}{m_b} \kappa_{cg} = 3 \text{ MeV}. \quad (50)$$

It follows the mass difference

$$\Lambda_b(J = 3/2) - \Lambda_b(J = 1/2) = 9 \text{ MeV}, \quad (51)$$

with nearly degenerate states because of the small value of κ_{bg} .

3.3 Mass splittings of flavor octet and decuplet hybrid baryons

We determine the masses and mass splittings of the flavor octet and decuplet hybrid baryons which contain two light quarks and one heavy quark i . If the hyperfine interaction V_{hyp} (18) would be absent, the *spin flavor* states **2****1**, **2****8**, **4****8** and **2****10** which form the **816** in (1), would be degenerate and would have the common mass E_{0i} . If V_{hyp} is present, the mass of each hybrid isospin multiplet is given by

$$E_i = E_{0i} + E_{hyp}. \quad (52)$$

Having found all values for κ_i , κ_g and κ_{ig} , we can calculate $E_{hyp} = \langle V_{hyp} \rangle$ from (18) for $\Lambda_s(E_s = 1405)$ and $\Lambda_c(E_c = 2593)$

$$E_{hyp}(\Lambda_s) = -291 \text{ MeV}; \quad E_{hyp}(\Lambda_c) = -191 \text{ MeV} \quad (53)$$

and find with (52) the mean hybrid baryon masses

$$E_{0s} = 1696 \text{ MeV}; \quad E_{0c} = 2784 \text{ MeV}. \quad (54)$$

We can now predict the masses of the strange and charmed flavor octet and decuplet hybrid baryons. From (22), (26) and tables 2 and 3 we calculate the interaction energy E_{hyp} and list the the resulting absolute masses E_i (52) of the flavor octet and decuplet in Tab. 5. We neglect a possible mixing of the hybrids with other states which carry the same quantum numbers since a theory of mixing with nearby ordinary octets and decuplets must be developed first. We also give the mass splittings ΔE_b for all the beauty hybrids.

4 Mass splittings in the mesonic sector

In Section 3 we have determined the value of the effective quark-gluon coupling $\kappa_g = 60$ MeV, which explains the $\Lambda_s(1405) - \Lambda_s(1520)$ splitting in the baryonic sector. As a matter of course, this value was obtained under the assumption that $\Lambda_s(1405)$ and $\Lambda_s(1520)$ are hybrid baryons. Although the found value also gives the correct splitting of $\Lambda_c(2625) - \Lambda_c(2593)$, an independent determination of κ_g is desirable. We want to stress that we cannot determine κ_g in our formalism independently of the hybrid assumption, since hybrids have not yet been discovered in experiment. However, theoretical mass predictions of hybrid mesons have been performed in the bag model [26]. In this section

Table 5: Predicted masses of the flavor decuplet and flavor octet hybrids, energies are given in MeV.

<i>spin flavor</i>	<i>J</i>	E_s	E_{hyp}	E_c	E_{hyp}	ΔE_b
$^4\mathbf{8}$ (I=1)	5/2	1809	113	2882	98	$E(5/2) - E(3/2) = 76$
	3/2	1689	-7	2796	12	$E(3/2) - E(1/2) = 46$
	1/2	1617	-79	2744	-40	
$^4\mathbf{8}$ (I=0)	5/2	1792	96	2847	63	$E(5/2) - E(3/2) = 123$
	3/2	1655	-41	2722	-62	$E(3/2) - E(1/2) = 73$
	1/2	1573	-123	2647	-137	
$^2\mathbf{8}$ (I=1)	3/2	1721	25	2844	60	$E(3/2) - E(1/2) = 87$
	1/2	1634	-62	2757	-27	
$^2\mathbf{8}$ (I=0)	3/2	1637	-59	2666	-118	$E(3/2) - E(1/2) = 5$
	1/2	1580	-166	2649	-135	
$^2\mathbf{10}$ (I=1)	3/2	1838	142	2884	100	$E(3/2) - E(1/2) = 83$
	1/2	1808	112	2813	29	

we will use the hybrid meson mass splittings calculated in [26], in order to obtain a value in the mesonic sector, denoted by κ_g^M , and relate it to the value we have found in the baryonic sector.

In our formalism the hyperfine interaction for hybrid mesons is given by

$$V_{hyp} = V_{q\bar{q}} + V_{qg} \quad (55)$$

where the quark-anti quark interaction is

$$V_{q\bar{q}} = -\kappa^M \mathbf{S}^q \cdot \mathbf{S}^{\bar{q}} \mathbf{F}^q \cdot \mathbf{F}^{\bar{q}}, \quad (56)$$

with κ^M the coupling strength between the quark and the anti quark inside the meson, which we determine in Section 4.1, and the quark-gluon interaction

$$V_{qg} = -\kappa_g^M \mathbf{S}^q \cdot \mathbf{S}^g \mathbf{F}^q \cdot \mathbf{F}^g - \kappa_g^M \mathbf{S}^{\bar{q}} \cdot \mathbf{S}^g \mathbf{F}^{\bar{q}} \cdot \mathbf{F}^g, \quad (57)$$

with κ_g^M the coupling strength between the quark (or anti quark) and the gluon inside the hybrid meson, which we determine in Section 4.2. Compared to the effective couplings κ found for baryons in Section 3, we expect that the values κ^M for mesons are somewhat larger. The reason is that more precise mass predictions can be made using interactions which include the bag radius r of the hadron [22, 26],

$$V_{qq'} = -\frac{\kappa}{r} \mathbf{S}^q \cdot \mathbf{S}^{q'} \mathbf{F}^q \cdot \mathbf{F}^{q'}, \quad (58)$$

an effect which we have neglected in this work so far due to simplicity. In the bag model [26] mesons are more spatially compact than baryons, with $r^M \approx 0.8 - 1.1$ fm and $r^B \approx 1.3 - 1.4$ fm and thus $r^B/r^M \approx 1.3 - 1.6$. We have to include this scaling if we want to relate the couplings from the mesonic sector to those of the baryonic sector, and take very approximately $\kappa^M \approx 1.6 \kappa$.

4.1 Quark-quark couplings for mesons

The quark-anti quark interaction (56) can be written as

$$V_{q\bar{q}} = -\kappa^M \frac{1}{4} \left[(\mathbf{S}^q + \mathbf{S}^{\bar{q}})^2 - (\mathbf{S}^q)^2 - (\mathbf{S}^{\bar{q}})^2 \right] \left[(\mathbf{F}^q + \mathbf{F}^{\bar{q}})^2 - (\mathbf{F}^q)^2 - (\mathbf{F}^{\bar{q}})^2 \right], \quad (59)$$

and the splitting of the $S = J = 1$ and the $S = J = 0$ meson multiplet is thus given by

$$E(J=1) - E(J=0) = \frac{4}{3} \kappa_i^M. \quad (60)$$

The effective couplings $\kappa^M, \kappa_s^M, \kappa_c^M, \kappa_b^M$ for mesons can now be determined, see table 6. Note that we obtain a more reasonable value for $\kappa^M = 372$ MeV if we take the predicted

Table 6: The mesons used to fix the κ_i^M

κ_i^M	meson	J	content	ΔM [MeV]	κ_i^M [MeV]	$\kappa_i = \kappa_i^M/1.6$ [MeV]
κ^M	$\rho(776)$	1	qq	638(496)	479(372)	299(233)
κ_s^M	$\pi(138(280))$	0				
	$K^*(894)$	1		398	299	187
	$K(496)$	0	qs			
κ_c^M	$D^*(2007)$	1		142	107	67
	$D(1865)$	0	qc			
κ_b^M	$B^*(5325)$	1		46	35	22
	$B(5279)$	0	qb			

mass of $\pi_{bag}(280)$ in the bag model [26], instead of the physical mass $\pi(138)$. In the limit of chirality conservation the π is the 'would be Goldstone boson' and thus has a light mass. The bag model cannot account for this effect, thus we will use $\kappa = 372$ MeV. Comparing the κ_i with the values obtained from baryon splittings, see Table 4 in Section 3.1, we find quite good agreement, since we included the size effect of the radii.

4.2 Mass splittings of hybrid mesons

We consider hybrid mesons consisting of two quarks q and \bar{q} which are light $q = u, d$. We will also allow that one of the quarks can be strange $q = s$. The two quarks form a color octet state with either $S = 0$ or $S = 1$ which couples to a TE gluon with $J^{PC} = 1^{+-}$. The $q\bar{q}$ state with $S = 0$ makes then a $J^{PC} = 1^{--}$ hybrid meson, and the $S = 1$ $q\bar{q}$ state makes three hybrid mesons with $J^{PC} = 0^{-+}, 1^{-+}, 2^{-+}$. Due to the hyperfine interaction (55) the degeneracy of these hybrid meson multiplets is lifted. We give our calculated values in Table 7.

The absolute masses of the non-strange hybrids ρ/ω and the strange hybrids K^* have been calculated in [26] for three different values of the ratio of the gluon TE and TM self energies, $C_{TE}/C_{TM} = \frac{1}{2}, 1, 2$. We obtain the same ordering of states for each of these ratios. We use the splittings of ρ/ω to determine κ_g^M and find $100 \text{ MeV} \lesssim \kappa_g^M \lesssim 140 \text{ MeV}$. Fitting the splittings of the strange hybrids K^* we obtain $60 \text{ MeV} \lesssim \kappa_{gs}^M \lesssim 130 \text{ MeV}$. These values of the coupling strengths are consistent with the results from ordinary meson splittings, since they have to obey (21)

$$\frac{\kappa^M}{\kappa_g^M} = \frac{\kappa_s^M}{\kappa_{gs}^M}, \quad (61)$$

Table 7: Summary table of hybrid meson interaction energies

J^{PC}	type	content	$\langle V_{q\bar{q}} \rangle$	$\langle V_{qg} \rangle$
1 ⁻⁻	ρ/ω	qq	$1/8\kappa^M$	0
1 ⁻⁻	K^*	qs	$1/8\kappa_s^M$	0
0 ⁻⁺	ρ/ω	qq	$-1/24\kappa^M$	$-3\kappa_q^M$
0 ⁻⁺	K^*	qs	$-1/24\kappa_s^M$	$-3/2(\kappa_q^M + \kappa_{qs}^M)$
1 ⁻⁺	ρ/ω	qq	$-1/24\kappa^M$	$-3/2\kappa_q^M$
1 ⁻⁺	K^*	qs	$-1/24\kappa_s^M$	$-3/4(\kappa_q^M + \kappa_{qs}^M)$
2 ⁻⁺	ρ/ω	qq	$-1/24\kappa^M$	$3/2\kappa_q^M$
2 ⁻⁺	K^*	qs	$-1/24\kappa_s^M$	$3/4(\kappa_q^M + \kappa_{qs}^M)$

which is quite well fulfilled for our values $\kappa^M = 370$ MeV, $\kappa_g^M = (120 \pm 20)$ MeV and $\kappa_s^M = 300$ MeV, $\kappa_{gs}^M = (95 \pm 35)$ MeV. Deducing from $100 \text{ MeV} \lesssim \kappa_g^M \lesssim 140 \text{ MeV}$ a value for the baryonic sector, we include the scaling $\kappa_g^M \approx 1.6 \kappa_g$, as discussed in the beginning of this section, and find $60 \text{ MeV} \lesssim \kappa_g \lesssim 90 \text{ MeV}$. This value is compatible with our result $\kappa_g = 60$ MeV obtained in (49), and adds credibility to the hypothesis that $\Lambda(1405)$ and $\Lambda(1520)$ are hybrid baryons.

5 The parity of $\Lambda(1405)$

As noted in the Introduction, the parity of $\Lambda(1405)$ has not been directly measured experimentally [2, 27, 28]. This is because the $\Lambda(1405)$ mass is below threshold in KN scattering, and it must be studied in $\Sigma\pi$ scattering, specifically in $Kp \rightarrow \Sigma\pi\pi\pi$ via $\Lambda^* \rightarrow \Sigma\pi$ followed by $\Sigma \rightarrow p\pi$, where $\Lambda(1405)$ is observed as a resonance in the Λ^* channel. Unfortunately, the dynamics of the Λ^* production in this process are such that it is produced with a small degree of polarization, so the interference between s - and p -wave final states which provides the sensitivity to the relative $\Lambda^* - \Sigma$ parity is small and the parity is therefore very difficult to measure [27]. In principle this might be rectified by an experiment such as $Kp \rightarrow \Sigma\pi\pi\gamma$, where there is at least the opportunity for more favorable production dynamics. Additionally, if the hybrid interpretation of the $\Lambda(1405)$ is correct, it might be *comparatively* copiously pair produced in J/Ψ decay. The conclusion of [2] that $\Lambda(1405)$ has negative parity is based on an indirect argument about the below-threshold line shape. Argumentation based on dynamical expectations to distinguish between s - and p -wave amplitudes is particularly dangerous because no known model fits the energy dependence of the data. As noted by Dalitz [2], the argument for negative parity is very weak and it is only adopted because there seemed no theoretical motivation to consider the parity to be positive.

The *lightest* hybrid flavor singlet baryon is likely to have even parity because the bag model predicts [15] the lightest $J^P = 1^{(-)}(TM)$ gluon mode is about 300 MeV heavier than the lightest $J^P = 1^{(+)}(TE)$ gluon mode. This is also corroborated by lattice calculations [29]. Therefore if the $\Lambda(1405)$ and $\Lambda(1520)$ are the lightest flavor singlet hybrid baryons they should have even parity. Their negative parity partners should have masses of about 1.7 GeV and 1.8 GeV. The experimental detection of such resonances would be difficult,

due to mixing with ordinary quark model states which are abundant in that mass range. The second possibility is that the $\Lambda(1405)$ and $\Lambda(1520)$ have odd parity and are *not* the lightest flavor singlet hybrid baryons. This would imply the existence of a lower mass pair of even parity states with mass about 1.1 GeV and 1.2 GeV! It is doubtful to us that such low energy states could be discovered. They would be below threshold in any baryon number $B = +1$, strangeness $S = -1$ meson-baryon state so their effect could only be observed as a resonance in $N\pi$, or possibly $\Lambda\gamma$ if the resonance mass is heavy enough. However the effect of a light strange even-parity hybrid resonance on a πN invariant mass plot or partial wave analysis would be extraordinarily small because it would be coupled only via weak interaction and only effect p -wave states. The most promising place to look might be in J/Ψ decays, where hybrid baryon pair production might be favored by OZI considerations; if kinematically allowed, $\Lambda\gamma$ would be the most promising final state.

It is astonishing to contemplate that one or more new strong-interaction-stable baryon states might exist, but we can see no compelling argument against it. Fortunately, this possibility has other implications which can be tested, as will be discussed in the next section. Lattice QCD calculations of masses of the positive parity flavor singlet baryon spectrum would be most illuminating in this context, and should be feasible. If they are consistent with the masses under discussion here (1.1 GeV - 1.4 GeV), every effort should be made to pursue the experimental searches suggested above. In the absence of additional information favoring this hypothesis, we favor the more conservative hypothesis that the $\Lambda(1405)$ and $\Lambda(1520)$ have positive parity.

6 A low lying dihyperon?

We have explored the ansatz that a uds in a color octet, flavor singlet state binds with a constituent gluon to produce the $\Lambda(1405)$. We have analyzed mass splittings between members of the various multiplets, but have made no absolute mass predictions. Lattice QCD would be the only trustworthy way to do this because *absolute* mass predictions of phenomenological models such as Skyrme, MIT bag or potential models are notoriously unreliable. To the best of our knowledge, no lattice calculation of hybrid baryon masses has yet been performed. However, glueball masses have been calculated on the lattice [30]. The lightest (pure) glueball is predicted to have a mass in the range 1.4 GeV - 1.7 GeV and there are good glueball candidates in this range. If the hybrid baryon ansatz is accepted for the $\Lambda(1405)$ and $\Lambda(1520)$, then the approximate coincidence of the $udsg$ and gg masses implies that a color octet state of three light quarks in a flavor singlet is approximately equivalent to a gluon as a dynamical system. This is not an outlandish idea as the dynamics of a hadronic bound state depends primarily on the color, mass and spin of the constituents. Thus a spatially-compact uds system in a flavor singlet state naturally resembles a gluon just having spin 1/2 rather than spin 1. Given the uncertainty discussed above as to whether or not the $\Lambda(1405)$ and $\Lambda(1520)$ are the lightest states, the uncertainty in “constituent” mass of this flavor-singlet, color-octet uds system is 300-500 MeV.

Pursuing the ansatz that a flavor singlet uds_8 system behaves much like a gluon, a combination of two uds_8 should be a glueball-like state with mass below ~ 2 GeV. This would be the H-dibaryon, which is an even parity six-quark state with spin $J = 0$ and isospin $I = 0$, baryon number $B = 2$ and strangeness $S = -2$. The H was predicted in 1977 by Jaffe [23] in a MIT bag model calculation, which estimated the mass to be

about 2150 MeV. Since then, many other H mass calculations have been performed, using Skyrme, quark cluster models, lattice calculations and instanton-based interactions [31] (for an overview see [32]). The resulting mass estimates range from less than 1.5 GeV to more than 2.23 GeV ($\Lambda\Lambda$ -threshold) which would make the H decay strongly. The wide range of the mass predictions for the H reflect the theoretical uncertainties in its existence and structure. If the $\Lambda(1405)$ is a hybrid baryon, we suggest that the mass of the H-dibaryon could lie in the 1.5-2 GeV mass region¹. If the H is lighter than two nucleons, an important concern would be the stability of nuclei against conversion over cosmological times, of pairs of nucleons to the H. Remarkably, this and other phenomenological difficulties with a stable or long-lived H are not fatal, if the H is sufficiently compact [17, 18, 19]. The discovery of a light H, especially if its mass were such that it was the ground state for $B = 2$, would be a stunning discovery in its own right. As a byproduct, it would lend strong support to the proposal that the $\Lambda(1405)$ is a hybrid. On the other hand a stable H is not a necessary consequence of the hypothesis of a strong attraction in the uds_8 system, so excluding it would not be sufficient to exclude the suggestion that $\Lambda(1405)$, etc, are hybrids. Lattice QCD calculations will be the most effective tool for that.

7 Summary and conclusions

We have explored the hypothesis that a very strong attraction in the uds_8 state may mean that the four isosinglet baryons $\Lambda_s(1405)$, $\Lambda_s(1520)$, $\Lambda_c(2593)$ and $\Lambda_c(2676)$ are hybrids. The observed mass splittings are consistent with the hybrid baryon hypothesis, resolving a persistent problem of the conventional identification as an orbital excitation of a 3-quark state. It is non-trivial that the ordering of states is $m_{J=3/2} > m_{J=1/2}$, as observed experimentally, because in the conventional $L = 1$ picture the $J = 3/2$ state is necessarily the lightest due to spin-orbit interactions. Assuming these states are flavor singlet hybrid baryons fixes the parameters of the quark-gluon hyperfine interaction. We have shown that the obtained quark-gluon coupling is consistent with calculations from the mesonic sector. In addition, this allows the mass splittings of the flavor octet and decuplet hybrid baryons to be predicted. A theory of mixing with nearby ordinary octets and decuplets must be developed before these predictions can be tested.

The best experimental test of our ansatz that the $\Lambda_s(1405)$, $\Lambda_s(1520)$, $\Lambda_c(2593)$ and $\Lambda_c(2676)$ are hybrids, is to determine whether they are parity doubled, with the odd parity partner about 300 MeV heavier than the even parity state. This scenario predicts either that the $\Lambda_s(1405)$, $\Lambda_s(1520)$, $\Lambda_c(2593)$ and $\Lambda_c(2676)$ are even parity, or that there are as-yet undiscovered even parity flavor singlet, strangeness -1 states at about 1.1 GeV and 1.2 GeV. The hybrid ansatz suggests, but does not predict, that the H-dibaryon mass may be as low as 1.5 GeV. Further work is needed to see if these possibilities can be excluded on observational grounds. In parallel, lattice QCD calculation of the masses of $udsg$ states and of the H-dibaryon mass will indicate whether this is a correct interpretation.

¹See [17] for a more refined mass estimation.

8 Acknowledgement

This work was supported in part by NSF-PHY-99-96173 and NSF-PHY-0101738.

A Casimir operators

Values for the $SU(2)$ and $SU(3)$ Casimir operators

$$C_2 = \mathbf{S} \cdot \mathbf{S} = \frac{1}{4} \sum_{m=1}^3 \sigma_m \sigma_m, \quad C_3 = \mathbf{F} \cdot \mathbf{F} = \frac{1}{4} \sum_{a=1}^8 \lambda_a \lambda_a, \quad (62)$$

are given by:

$SU(2)$	$\begin{array}{ c c c c c c }\hline dim & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \\ \hline C_2 & 0 & 3/4 & 2 & 15/4 & 6 \\ \hline\end{array}$	$SU(3)$	$\begin{array}{ c c c c c c }\hline dim & \mathbf{1} & \mathbf{3} & \mathbf{6} & \mathbf{8} & \mathbf{10} \\ \hline C_3 & 0 & 4/3 & 10/3 & 3 & 6 \\ \hline\end{array}$
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B Mixed symmetry functions

Table 8: The definitions of mixed symmetric and mixed antisymmetric wave functions φ_{MS} and φ_{MA} , respectively. The table is taken from [15].

label	φ_{MS}	φ_{MA}
P	$\frac{1}{\sqrt{6}} [(ud + du)u - 2uud]$	$\frac{1}{\sqrt{2}}(ud - du)u$
N	$-\frac{1}{\sqrt{6}} [(ud + du)d - 2ddu]$	$\frac{1}{\sqrt{2}}(ud - du)d$
Σ^+	$\frac{1}{\sqrt{6}} [(us + su)u - 2uus]$	$\frac{1}{\sqrt{2}}(us - su)u$
Σ^0	$\frac{1}{\sqrt{6}} \left[s \left(\frac{du+ud}{\sqrt{2}} \right) + \left(\frac{dsu+usd}{\sqrt{2}} \right) - 2 \left(\frac{du+ud}{\sqrt{2}} \right) s \right]$	$\frac{1}{\sqrt{2}} \left[\left(\frac{dsu+usd}{\sqrt{2}} \right) - s \left(\frac{ud+du}{\sqrt{2}} \right) \right]$
Σ^-	$\frac{1}{\sqrt{6}} [(ds + sd)d - 2dds]$	$\frac{1}{\sqrt{2}}(ds - sd)d$
Λ^0	$\frac{1}{\sqrt{2}} \left[\left(\frac{dsu-usd}{\sqrt{2}} \right) + \frac{s(du-ud)}{\sqrt{2}} \right]$	$\frac{1}{\sqrt{6}} \left[\frac{s(du-ud)}{\sqrt{2}} + \frac{usd-dsu}{\sqrt{2}} - \frac{2(du-ud)s}{\sqrt{2}} \right]$
Ξ^-	$-\frac{1}{\sqrt{6}} [(ds + sd)s - 2ssd]$	$\frac{1}{\sqrt{2}}(ds - sd)s$
Ξ^0	$-\frac{1}{\sqrt{6}} [(us + su)s - 2ssu]$	$\frac{1}{\sqrt{2}}(us - su)s$

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